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Phonon-assisted tunnelling in a magnetic field: applications to hopping conductivity, tunnel junction and the quantum Hall effect

A V Khaetskii† and K A Matveev‡

† Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany and
Institute of Microelectronics Technology and High Purity Materials, Russian Academy of Sciences, 142432 Chernogolovka, Moscow District, Russia
‡ Institute of Solid State Physics, USSR Academy of Sciences, 142432 Chernogolovka, Moscow District, Russia

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Abstract. Electron tunnelling in a transverse magnetic field is studied taking into account the electron–phonon interaction. When scattered on a phonon, the electron shifts the oscillator centre in the tunnelling direction, which decreases the field-related magnetic barrier and increases the probabilty of tunnelling considerably. For tunnelling over large distances, the most effective processes involve multiple scattering by phonons. An expression has been found for the resistivity of the Miller–Abrahams network element for hopping conductivity with regard to magnetic-barrier suppression upon scattering by phonons. The exponential temperature and voltage dependences of the tunnel junction conductance in a magnetic field parallel to a dielectric interlayer have been obtained. The contribution associated with the tunnelling processes involving scattering by two phonons to the relaxation of edge-state populations in the quantum Hall effect regime has been found.

1. Introduction

Investigation of many problems in solid state physics requires an understanding of the process of electron tunnelling in a magnetic field. Among such problems are: the study of the current flow in a tunnel junction placed in the magnetic field; hopping magnetoresistance of semiconductors; and tunnelling between the edge states in the quantum Hall effect regime. The present paper is concerned with the processes of single- or multiphonon-assisted electron tunnelling in the direction perpendicular to the magnetic field. It is shown that scattering by phonons facilitates tunnelling considerably and gives rise to an exponential temperature dependence of the tunnelling probability.

The influence of the transverse magnetic field on the wavefunction of the impurity-bound electron's ground state manifests itself in the change of the asymptotic behaviour $\Psi \propto \exp(-\rho/a)$ to

$$\Psi(\rho) \propto \exp(-\rho^2/4\lambda^2) \tag{1}$$

where $\lambda = \sqrt{c\hbar/|e|H}$ is the magnetic length, and $\rho = (x, y)$; the electrons are assumed to be two-dimensional with the magnetic field H directed along the z axis. This fact may be interpreted as the result of the occurrence of an additional potential barrier, $V_H = \hbar^2\rho^2/8m\lambda^4$, called the magnetic barrier [1]. In contrast to the conventional potential, V_H is not fixed in space, i.e. if the electron is scattered with a transfer of momentum $\hbar q$, then the origin of the magnetic barrier is shifted within a distance $\lambda^2 q$ in the direction $[qH]$. In the case of multiple scattering with momentum transfer perpendicular to the tunnelling direction, the magnetic barrier stops increasing monotonically with increasing ρ. This leads to the logarithm of the tunnelling probability depending linearly on distance d:

$$\ln W \simeq -d/b \tag{2}$$

where b is the characteristic length, which depends on $\mathcal{H}$ and the scattering intensity. This has been shown for the case of scattering by impurities (or the crystal boundary) [2–4]. In the present paper it is demonstrated that, at not too low temperatures, the magnetic barrier may be suppressed, even in the absence of impurities due to scattering by phonons. In contrast to scattering by static defects, the momentum transfer to the phonon is inevitably accompanied by energy transfer. In particular this leads to the above-mentioned strong temperature dependence of the tunnelling probability.

Section 2 considers single- and multiphonon-assisted electron hopping between two localized states. The probability of such an electron hop has been calculated as a function of intercentre distance d, level energy difference $E_i - E_f$ and temperature (for sufficiently strong magnetic fields, such that the magnetic length λ is much less than d, throughout). If the difference in energy between the initial and final states is fairly great:

$$E_i - E_f > \hbar s d/\lambda^2 \tag{3}$$

then acoustic multiphonon-assisted processes involving the shift of the oscillator centre† by the distance d are possible even at zero temperature (s is the sound velocity). The probability of tunnelling with emission of an arbitrary number of phonons has the form

$$W \propto \sum_n \alpha^{2n}\left[\exp\left\{-\frac{1}{2\lambda^2}\left(\frac{d}{n+1}\right)^2\right\}\right]^{n+1} = \sum_n \alpha^{2n}\exp\left(-\frac{d^2}{2\lambda^2(n+1)}\right). \tag{4}$$

Here α is the small dimensionless constant of the electron–phonon interaction. Equation (4) takes into account the fact that in the n-phonon process the electron covers distance $d/(n+1)$ between two sequential acts of phonon emission. Calculation of summation (4) yields (2) with characteristic length $b = \lambda/(2\ln^{1/2}(1/\alpha))$. The total momentum transferred to the emitted phonons cannot exceed the value $\Delta p = (E_i - E_f)/s$ and, hence if condition (3) is not fulfilled, then the total shift of the oscillator centre, $\Delta x = \lambda^2\Delta p/\hbar$, appears to be smaller than d (here it is assumed that $E_i > E_f$). Therefore, at $T = 0$ the tunnelling probability appears to be

† Hereafter, instead of the term 'the origin of the magnetic barrier', we adopt the more commonly used term 'the oscillator centre'.

exponentially smaller than the value yielded by (4) by parameter $(d-\Delta x)^2/\lambda^2$. Further increase of temperature initiates the tunnelling processes accompanied by both emission and absorption of phonons. Such processes involve transfer of a larger momentum to the phonon system and, hence, a larger total shift of the oscillator centre Δx†. Thus, the tunnelling probability increases exponentially with temperature.

The expression found in section 2 for the probability of hopping between two localized states involving an arbitrary number of phonons is used to calculate the resistivity of an element of the Miller–Abrahams network [1] for two-dimensional hopping conductivity in a transverse magnetic field. Under certain conditions the effects related to the shift of the oscillator centre may show up in hopping magnetoresistance.

In section 3 we investigate the passage of current in the tunnel junction in the presence of a magnetic field parallel to the plane of the dielectric interlayer. The contact leads are assumed to contain impurities; on the contrary, the interlayer is assumed to be free of impurities and its thickness d much larger than the magnetic length λ. The dependence of differential conductance on temperature and on the voltage applied to the contact, $G(V,T)$, has been found. At $T = 0$ and $V = 0$, the conductance is exponentially small, i.e. $G \propto \exp(-d^2/2\lambda^2)$, and determined by the processes of elastic electron tunnelling. With increasing voltage on the contact (and at $T = 0$) the conductance increases exponentially, beginning with the characteristic value $V \sim V_d = \hbar s/ed$ due to initiation of the tunnelling processes with phonon emission. At $V > V^* = \hbar s d/e\lambda^2$, the processes involving a shift of the oscillator centre, $\Delta x = d$, are allowed (see also (3)) and the conductance is saturated and has the value

$$G \propto \exp(-(2d/\lambda)\ \ln^{1/2}1/\alpha). \tag{5}$$

The increase in temperature at $V = 0$ also results in an exponential increase of conductance, but, in contrast to the case for $T = 0$ and $V \neq 0$, this is due to the tunnelling processes involving both emission and absorption of phonons. The temperature dependence of conductance originates at threshold temperature $T_2 = \hbar s/2d$ and terminates at $T \sim (\hbar s/\lambda)\ln^{1/2}1/\alpha$ when the conductance reaches the value in (5). It is of interest to note that at voltages in the interval $V_d < V < V^*$ the conductance $G(V,T)$ exhibits a strong temperature dependence, even at $T \ll eV$.

Section 4 considers the possibility of observing the multiphonon-assisted tunnelling effect in 2D ballistic structures in a magnetic field. In particular, the correction factor to the quantized Hall resistance value, which is exponentially dependent on temperature, should be observed due to the phonon-enhanced tunnelling between the edge states.

2. The probability of a single hop between localized states

The Hamiltonian of the 2D electron in a perpendicular magnetic field in the presence of two impurity centres takes the form

$$\hat{\mathcal{H}}_e = (1/2m)(-i\hbar\nabla - (e/c)A)^2 + v_1(\rho-\rho_1) + v_2(\rho-\rho_2) \tag{6}$$

† To this end, it is certainly essential that the momenta of the absorbed and emitted phonons are opposite in direction.

where $\rho_j = (x_j, y_j)$; the impurities are described by the small-radius potentials $v_1(\rho)$ and $v_2(\rho)$; the impurity spacing $|\rho_1 - \rho_2| = d \gg \lambda$. Let us choose the vector potential in the Landau gauge: $A_y = Hx, A_x = A_z = 0$. The wavefunctions of the states localized on impurities 1,2 may be written as

$$\Psi_1 = \varphi_1 + \beta_1\varphi_2 \qquad \Psi_2 = \varphi_2 + \beta_2\varphi_1 \tag{7}$$

$$\beta_1 = \langle\varphi_2|v_2|\varphi_1\rangle/(E_1 - E_2) \qquad \beta_2 = \langle\varphi_1|v_1|\varphi_2\rangle/(E_2 - E_1) \tag{8}$$

where φ_j, E_j are the wavefunction and energy of the electron localized on impurity j in the absence of the other impurity ($j = 1, 2$); it is assumed that $\hbar\omega_c/2 - E_j \ll \hbar\omega_c$, where $\omega_c = |e|H/mc$ is the cyclotron freqency. The asymptotics of the wavefunction φ_j has the form

$$\varphi_j(\rho) = (1/\sqrt{2\pi\lambda^2})\exp(-|\rho - \rho_j|^2/4\lambda^2) \times \exp\{-(\mathrm{i}/2\lambda^2)(xy + x_j y - y_j x)\} \qquad |\rho - \rho_j| \gg \lambda. \tag{9}$$

When writing the eigenfunctions of the Hamiltonian (6) in the form of (7) and (8), β_1 and β_2 were assumed to be small. This condition is fulfilled at sufficiently large values of d since, as seen from (9), the values of β_j are exponentially small:

$$\beta_j \propto \exp(-d^2/4\lambda^2). \tag{10}$$

Given the electron–phonon interaction, there are transitions between states Ψ_1 and Ψ_2 with emission or absorption of phonons. If we assume that electrons interact only with acoustic phonons, and the electron–phonon interaction is described by the deformation potential, then:

$$\hat{\mathcal{H}}_{\text{e-ph}} = \mathrm{i}\Lambda\sum_q \sqrt{\hbar q/2V_0\rho_0 s}(\exp(\mathrm{i}q\cdot r)b_q - \exp(-\mathrm{i}q\cdot r)b_q^+). \tag{11}$$

Here $r = (\rho, z)$, Λ is the deformation potential constant, b_q is the annihilation operator of the phonon with wavevector q, s is the velocity of the longitudinal sound, ρ_0 is the material density and V_0 is the sample volume. The probability of electron transition from state Ψ_1 to state Ψ_2 with absorption of m phonons and emission of l phonons may be written as

$$W_{1\to 2}^{(m,l)} = \frac{2\pi}{\hbar}\sum_{\{q\},\{q'\}} |\langle\Psi_2; (N_{q_1} - 1), \ldots, (N_{q_m} - 1), (N_{q'_1} + 1), \ldots, (N_{q'_l} + 1)|\hat{T}|\Psi_1; N_{q_1}, \ldots, N_{q_m}, N_{q'_1}, \ldots, N_{q'_l}\rangle|^2 \times \delta(E_1 - E_2 + E_{\text{abs}} - E_{\text{em}}). \tag{12}$$

Here N_q is the filling number of phonon state q, and E_{abs} and E_{em} are the total energies of absorbed and emitted phonons:

$$E_{\text{abs}} = \sum_{i=1}^{m} \hbar s|q_i| \qquad E_{\text{em}} = \sum_{j=1}^{l} \hbar s|q'_j|.$$

The summation in (12) is made over all the possible values of the wavevectors of absorbed phonons $q_1, \ldots, q_m$ and emitted phonons $q'_1, \ldots, q'_l$. Scattering operator $\hat{T}$ may be represented as a standard series of the perturbation theory with respect to the operator of the electron–phonon interaction (11):

$$\hat{T} = \hat{\mathcal{H}}_{\mathrm{e-ph}} + \hat{\mathcal{H}}_{\mathrm{e-ph}} \frac{1}{E_1 - \hat{\mathcal{H}}_{\mathrm{e}} - \hat{\mathcal{H}}_{\mathrm{ph}}} \hat{\mathcal{H}}_{\mathrm{e-ph}} + \ldots \tag{13}$$

$$\hat{\mathcal{H}}_{\mathrm{ph}} = \sum_{q} \hbar s |q| b_q^+ b_q . \tag{14}$$

The first term of series (13) corresponds to single-phonon-assisted tunnelling. Processes either with absorption or emission of a phonon are allowed depending on the relationship between E_1 and E_2. For instance, in the case of $E_1 > E_2$ we have

$$W_{1\to 2}^{(0,1)} = \frac{\pi \Lambda^2}{V_0 \rho_0 s} \frac{(E_1 - E_2)}{\hbar s} [N(E_1 - E_2) + 1] \times \sum_{q'} \left| \int \Psi_2^*(x, y) \mathrm{e}^{-\mathrm{i}(q'_x + q'_y y)} \Psi_1(x, y) \mathrm{d}x \, \mathrm{d}y \right|^2 \delta(E_1 - E_2 - \hbar s q') \tag{15}$$

where $N(E)$ is the equilibrium Planck phonon distribution function. Substitution of functions (7) into (15) yields two types of integrals, namely,

$$J_1 = \int \mathrm{d}^2\rho \, \varphi_2^* \exp(-\mathrm{i}q'\rho)\varphi_1 \qquad J_2 = \beta_2^* \int \mathrm{d}^2\rho \, \varphi_1^* \exp(-\mathrm{i}q'\rho)\varphi_1 .$$

In contrast to the case $\mathcal{H} = 0$ when $J_2 \gg J_1$ [1], calculations in the limit $\lambda \ll d$ yield $J_1 \gg J_2$. As a result, for the probability of hopping we find from (15)

$$W_{1\to 2}^{(0,1)} = \Lambda^2 (N+1) x_0^2 / 4\pi\hbar\rho_0 s^2 \lambda^4 \sqrt{d(d - 2x_0)} \times \exp\{-(x_0^2/2\lambda^2) - (d - x_0)^2/2\lambda^2\} \tag{16}$$

$$x_0 = \lambda^2 (E_1 - E_2)/\hbar s$$

Quantity x_0 represents the shift of the oscillator centre upon emission of a phonon with energy $E_1 - E_2$. Equation (16) is valid at $x_0 \gg \lambda^2/d$, $d - 2x_0 \gg \lambda$, therewith the momentum of the emitted phonon is directed almost perpendicular to the direction of tunnelling. Probability $W_{1\to 2}^{(0,1)}$ increases exponentially with increasing $E_1 - E_2$ owing to the increase in the shift of the oscillator centre x_0 on phonon emission. The probability of the reverse transition, $W_{2\to 1}^{(1,0)}$, is obtained from (16) by substitution of $(N+1)$ for N.

Let us show that under certain conditions multiphonon-assisted processes make an exponentially larger contribution to the tunnelling probability than single-phonon processes (see (16)). For instance, two-phonon contributions $W_{1\to 2}^{(0,2)}$ and $W_{1\to 2}^{(1,1)}$ determined by the second term of series (13) take the form

$$W_{1\to 2}^{(0,2)} = \frac{\sqrt{\pi}\Lambda^4 x_0^4}{(E_1 - \frac{\hbar\omega_c}{2})^2 4(4\pi)^3 \rho_0^2 s^3 \lambda^9 \sqrt{d(d-x_0)}(d-\frac{3}{2}x_0)} \times \exp\left\{-\frac{x_0^2}{4\lambda^2} - \frac{(d-x_0)^2}{2\lambda^2}\right\} \tag{17}$$

$$W_{1\to 2}^{(1,1)} \simeq \frac{\Lambda^4 d^3}{162(2\pi)^{5/2}\rho_0^2 s^3 L_T \lambda^9} \frac{1}{(E_1 - \frac{\hbar\omega_c}{2})^2} \times \exp\left\{-\frac{d^2}{2\lambda^2} + \frac{(d-\frac{L_T}{2})^2}{3\lambda^2} + \frac{x_0}{2\lambda^2}(L_T - \frac{x_0}{2})\right\} \tag{18}$$

where $L_T \equiv \hbar s/T$. Equation (17) is valid at $x_0 \gg \lambda^2/d$, $(2/3)d - x_0 \gg \lambda$. Equation (18) is valid to an exponential accuracy at $x_0 < L_T < 2d - 3x_0$; the pre-exponential in (18) depends in a complex way on x_0, L_T and d, the form presented above being valid within the limit $x_0 \ll L_T \ll d$.

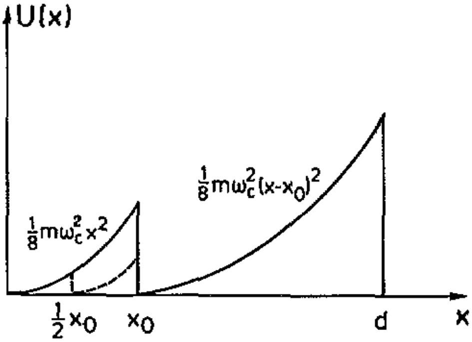

Figure 1. Effective magnetic barrier for tunnelling with the emission of one phonon (solid line). In transition to two-phonon processes the barrier is changed in the interval $(0, x_0)$ (dashed line).

Equation (17) yields the tunnelling probability with the emission of two phonons. The exponential factors in (16) and (17) for probabilities of single- and two-phonon processes differ in the numerical coefficient of the first component. This may be interpreted as follows. The exponent in (16) may be calculated as the quasiclassical probability of tunnelling through the barrier (figure 1). The barrier release at point x_0 is associated with emission of a phonon with momentum $\hbar x_0/\lambda^2$ in the direction perpendicular to the direction of tunnelling. For two-phonon emission the total shift of the oscillator centre also does not exceed x_0 since the total phonon energy is limited by $E_1 - E_2$. However, owing to the two-fold release of the magnetic barrier, (dashed line, figure 1), its effective height is reduced and the exponential multiplier in (17) appears to be larger than in (16) by a factor of $\exp(x_0^2/4\lambda^2)$.

For the processes of tunnelling with absorption of one phonon and emission of another phonon (the probability of which is described by (18)), the total shift of the oscillator centre is not limited by $\lambda^2(E_1 - E_2)/\hbar s$ since the energy conservation law now fixes the phonon energy difference rather than the phonon energy sum. The increase in the total shift of the oscillator centre (proportional to the total momentum transferred to the phonons) brings about a more effective suppression of the magnetic barrier and hence a significant increase in the tunnelling probability. Yet, at low temperatures the probability of the absorption of a phonon with a large momentum is exponentially small, the competition between the two factors leading to probability (18). Note that processes involving both emission and absorption of phonons result in an exponential gain in the tunnelling probability compared to single-phonon processes even when $(E_1 - E_2) \to 0$ (in contrast to processes with phonon emission only). For instance, at sufficiently high temperature (when $L_T \ll d$), $W_{1\to 2}^{(1,1)} \propto \exp(-d^2/6\lambda^2)$.

The exponential increase in the tunnelling probability in transition from single- to double-phonon processes illustrates the general fact noted in the introduction that multiple scattering leads to a more effective suppression of the magnetic barrier. Therefore, it is useful to obtain an expression for the probability of tunnelling involving an arbitrary number of phonons. The probability of a transition with absorption of m phonons and emission of l phonons is obtained by substituting the $(m+l)$th term of series (13) into (12) and has the form

$$W_{1\to 2}^{(m,l)} \propto \Lambda^{2(m+l)} \sum_{q,q'} \exp\left\{ -\sum_{i=1}^{m} \frac{\lambda^2 q_i^2}{2} - \sum_{j=1}^{l} \frac{\lambda^2 q_j'^2}{2} - \frac{1}{2\lambda^2}\left[(x_1 - x_2 + \lambda^2 Q_y)^2 + (y_1 - y_2 - \lambda^2 Q_x)^2\right] - \frac{E_{\mathrm{abs}}}{T} \right\} \delta(E_1 - E_2 + E_{\mathrm{abs}} - E_{\mathrm{em}}). \tag{19}$$

where $\hbar Q$ is the total momentum transferred to phonons:

$$Q = \sum_{j=1}^{l} q_j' - \sum_{i=1}^{m} q_i$$

and in (19) all the phonon energies are assumed to be greater than the temperature. The main contribution to the hopping probability is made by phonons with small q_z, which is taken into account in (19).

If the temperature is equal to zero, only processes with phonon emission are possible and in (19) it should be assumed that $m = 0$ and $E_{\mathrm{abs}} = 0$. The main contribution to the integral over q_j' is made by the processes for which $q_1' \simeq q_2' \ldots \simeq q_l' \simeq x_0/\lambda^2 l$, the momenta of all the phonons directed approximately along vector $\boldsymbol{H} \times (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$, i.e. perpendicular to the tunnelling direction. Hence, the probability of tunnelling with the emission of an arbitrary number of phonons may be estimated as follows:

$$W_{1-2} = \sum_{l=1}^{\infty} W_{1-2}^{(0,l)} \propto \sum_{l=1}^{\infty} \alpha^{2l} \exp\left\{ -\frac{x_0^2}{2\lambda^2 l} - \frac{1}{2\lambda^2}(d - x_0)^2 \right\}. \tag{20}$$

The dimensionless constant α is proportional to the deformation potential Λ. Quantity α^2 may be estimated† as the pre-exponent ratio in (17) and (16):

$$\alpha^2 \simeq \pi^{1/2} \hbar \Lambda^2 / 4(4\pi)^2 \rho_0 s \lambda^4 E^2$$

where it was assumed that $d \simeq x_0 \simeq \lambda$. At $\Lambda = 7$ eV, $\rho_0 \simeq 5$g cm^{-3}, $s = 5 \times 10^5$ cm s^{-1}, $\lambda = 10^{-6}$ cm and for binding energy $E = 0.1\hbar\omega_c$ we get $\alpha^2 \simeq 10^{-3}$–10^{-4}. At small values of the energy level difference, when $x_0 < \lambda \ln^{3/2}(1/\alpha)$, the

† Our following results depend only on $\ln(1/\alpha)$. Hence, the strict definition of α (which cannot be given in principle) is not necessary.

value of summation (20) is determined by one of its terms; in the opposite limit the summation may be substituted by the integral over l, following which we find

$$W_{1\to 2} \propto \exp\{-2(x_0/\lambda)\ln^{1/2}(1/\alpha) - (1/2\lambda^2)(d-x_0)^2\}$$
$$\text{for } \lambda \ln^{3/2}(1/\alpha) \ll x_0 \leqslant d. \tag{21}$$

At $x_0 > d$ the probability of tunnelling ceases to increase exponentially and

$$W_{1\to 2} \propto \exp(-(2d/\lambda)\ln^{1/2}(1/\alpha)). \tag{22}$$

At sufficiently high temperatures, such that $(d - x_0) - L_T/2 \gg \lambda \ln^{1/2}(1/\alpha)$, the most effective processes involve both emission and absorption of a large number of phonons. In this case the tunnelling probability should be calculated by summing (19) over m and l. The calculation is similar to that made above for $m = 0$ and yields

$$W_{1\to 2} \propto \exp\left\{-\frac{d}{\lambda}\left(\frac{L_T}{2\lambda} + 2\ln^{1/2}\frac{1}{\alpha}\right) + \frac{L_T^2}{8\lambda^2} + \frac{L_T}{\lambda}\ln^{1/2}\frac{1}{\alpha} + \frac{(E_1 - E_2)}{2T}\right\} \tag{23}$$

As was noted in the introduction, multiphonon scattering causes suppression of the component in the exponential factor (23) which is quadratic in d/λ. When deriving (23), it was assumed that the energy of each absorbed phonon is large compared to the temperature. For this reason, (23) is valid at $T \ll (\hbar s/\lambda)\ln^{1/2}(1/\alpha)$.

So far in this section we have considered the case when $E_1 > E_2$. The formulae for the tunnelling probabilities when $E_2 > E_1$ may be obtained using the general relationship for the probabilities of direct and reverse transitions.

$$W_{1\to 2} = W_{2\to 1}\exp((E_1 - E_2)/T). \tag{24}$$

The expressions for the tunnelling probability given above may be used in studies of two-dimensional hopping conductivity in a transverse magnetic field. Here we restrict ourselves to calculation of the conductivity R_{12}^{-1} of the element of the Miller–Abrahams network (see, for example, section 4.2 of reference [1]). Following the calculations made in [1] and using (24) we obtain

$$R_{12}^{-1} \propto W_{1\to 2}\exp(-(E_1 - E_2)/2T - (|E_1 - \mu| + |E_2 - \mu|)/2T). \tag{25}$$

Here μ is the chemical potential and one of the expressions (16–18), (21–23) should be substituted for tunnelling probability $W_{1\to 2}$. For instance, for single-phonon processes, from (16) and (25) we find

$$R_{12}^{-1} \propto \exp(-(x_0^2/2\lambda^2) - (d-x_0)^2/2\lambda^2)$$
$$\times \exp(-(|E_1 - E_2| + |E_1 - \mu| + |E_2 - \mu|)/2T). \tag{26}$$

Note that, unlike the formula used previously [6], (26) takes into account the shift of the oscillator centre by $x_0 = \lambda^2|E_1 - E_2|/\hbar s$ upon emission (or absorption) of a single phonon. This is likely to be most pronounced in hopping magnetoresistance in the region of intermediate magnetic fields ($a \ll \lambda \ll \sqrt{Ra}$ where a is the Bohr impurity radius and R is the mean impurity spacing).

Comparison between (16) and (18) shows that double-phonon processes make a greater contribution to the tunnelling probability than single-phonon ones at sufficiently high temperature

$$L_T < 2d - 3x_0 - 2\sqrt{6}\lambda \ln^{1/2}(1/\alpha). \tag{27}$$

Hence, to observe multiphonon effects, it is essential that parameter d/λ should be fairly great (e.g. $d/\lambda > \sqrt{6}\ln^{1/2}(1/\alpha)$), which is hardly realized in the known experiments on hopping magnetoresistance. The appropriate condition, however, may be realized in a specially prepared tunnel junction.

3. Tunnel junction in a transverse magnetic field

Let us consider the tunnel junction of two metallic conductors separated by a dielectric interlayer. We will calculate the current passing in such a junction as a function of temperature T and applied voltage V in the presence of a magnetic field $\boldsymbol{H}$ parallel to the interlayer plane. Assume that the interlayer thickness d is large and its related energy barrier U_0 is low so that the condition $m\omega_c^2 d^2 \gg U_0$† is fulfilled; this means that the exponential suppression of the tunnelling current is mainly caused by electrons overcoming the magnetic barrier ($d \gg \lambda$). In [3] it was shown that the presence of impurities in the interlayer facilitates tunnelling considerably due to the release of the magnetic barrier on subbarrier scattering. As distinct from [3], we assume here that the interlayer is free of impurities but that the leads contain a random potential. Below it will be shown that the tunnelling current is mainly related to the processes involving absorption and emission of phonons. Yet, first let us consider the elastic tunnelling (phonon-free) processes that are important at fairly low temperatures and voltages.

3.1. *Dependence of elastic conductance on interlayer thickness*

Let us describe the tunnel junction by means of the model Hamiltonian

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\text{imp}} \tag{28}$$

$$\hat{\mathcal{H}}_0 = (1/2m)(-i\hbar\nabla - (e/c)\boldsymbol{A})^2 + U_0\theta(x)\theta(d-x) \tag{29}$$

$$\hat{\mathcal{H}}_{\text{imp}} = V_1(\boldsymbol{r})\theta(-x) + V_2(\boldsymbol{r})\theta(x-d) \qquad \boldsymbol{r} = (\rho, z) \tag{30}$$

where $\theta(x)$ is the Heaviside function and the random potentials in the leads, $V_{1,2}$, are assumed to be delta-correlated:

$$\langle V_1(\boldsymbol{r})V_1(\boldsymbol{r}')\rangle = \gamma_1\delta(\boldsymbol{r}-\boldsymbol{r}') \qquad \langle V_2(\boldsymbol{r})V_2(\boldsymbol{r}')\rangle = \gamma_2\delta(\boldsymbol{r}-\boldsymbol{r}'). \tag{31}$$

† This condition can hardly be fulfilled in conventional MIM tunnel junctions. However, we suppose that the desired junction may be realized in a silicon MOSFET where the height of the barrier is adjusted by the gate voltage. Another possibility is to make a barrier of pure GaAs and the leads of heavily doped (degenerate) GaAs.

When choosing the vector potential in the Landau gauge, $A = (0, Hx, 0)$, the eigenstates of Hamiltonian $\hat{\mathcal{H}}_0$ may be classified in accordance with coordinate $X = \lambda^2 k_y$ of the magnetic oscillator centre. Let us relate the states corresponding to $X < d/2$ to the left lead and the states with $X > d/2$ to the right lead. Assume that the Fermi energy is smaller than the barrier height so that only states with $X < 0$ and $X > d$, are filled. Since the coordinate of the oscillator centre is the integral of motion, then in the absence of perturbation $\hat{\mathcal{H}}_{\rm imp}$ the current is equal to zero. In order to determine the dependence of conductance on interlayer thickness in the presence of an impurity potential in the leads, we calculate the mean square of the amplitudes of transitions from state $X_{\rm L}$ in the left lead to state $X_{\rm R}$ in the right lead in the first two orders of the perturbation theory in $\hat{\mathcal{H}}_{\rm imp}$:

$$\langle |T_{\rm L\to R}|^2 \rangle = \int {\rm d}^3 r \, |\psi_{\rm L}(r)|^2 |\psi_{\rm R}(r)|^2 \{\gamma_1 \theta(-x) + \gamma_2 \theta(x-d)\}$$

$$+ \gamma_1\gamma_2 \int {\rm d}^3 r \, {\rm d}^3 r' \, |\psi_{\rm L}(r)|^2 \theta(-x) |g(r, r')|^2 \theta(x'-d) |\psi_{\rm R}(r')|^2 . \quad (32)$$

Here $\psi_{\rm L}$ and $\psi_{\rm R}$ are the wavefunctions of states with oscillator centres $X_{\rm L}$ and $X_{\rm R}$; $g(r, r')$ is the Green function of the Schrödinger equation with Hamiltonian $\hat{\mathcal{H}}_0$ (29); and the angular brackets denote averaging over the impurity potential. Considering in (32) the asymptotic behaviour

$$|\psi_{\rm L,R}|^2 \propto \exp(-(x - X_{\rm L,R})^2/\lambda^2) \quad (33)$$

$$|g(r, r')|^2 \propto \exp(-|\rho - \rho'|^2/2\lambda^2) \quad (34)$$

for elastic conductance we obtain

$$G_{\rm el} = \xi(\gamma_1 + \gamma_2) {\rm e}^{-d^2/\lambda^2} + \xi' \gamma_1 \gamma_2 {\rm e}^{-d^2/2\lambda^2} . \quad (35)$$

Dimension coefficients ξ and ξ' take into account the densities of the states in the leads and are independent of interlayer thickness d. As seen from (35), two types of the exponential dependence of elastic conductance on d are possible. If the impurity potential is weak ($\gamma_1, \gamma_2 \to 0$), then the conductance is determined by the first term in (35) corresponding to such scattering processes when, for instance, an electron is scattered from the left lead to the right one on the right-lead impurity (see the first component in (32)). In this case the thickness dependence, $G_{\rm el} \propto \exp(-d^2/\lambda^2)$, is determined by the asymptotic behaviour of the wavefunctions (33). If the impurity potential is not too weak, the thickness dependence of conductance is determined by the second term in (35) which corresponds to scattering processes on a pair of impurities located in the opposite leads of the junction. In this case dependence $G_{\rm el} \propto \exp(-d^2/2\lambda^2)$ is determined by the square of the modulus of the Green function (34) calculated between two neighbouring points in the opposite leads†. It is clear that at sufficiently high values of d/λ the thickness dependence of conductance (35) is determined by the second term.

Formulae (32), (35) take into account only the first two orders of the expansion of the tunnelling amplitude in terms of the impurity potential. If the latter is not small, account should also be taken of higher orders which, as can be seen, do not change the exponential dependence of conductance on thickness, $G_{\rm el} \propto \exp(-d^2/2\lambda^2)$.

† A similar relationship between conductance and the Green function was discussed in [3] for the case of scattering by impurities in the interlayer.

3.2. Conductance and the current–voltage characteristic in view of subbarrier scattering by phonons

In this section we calculate the differential conductance of the tunnel junction in a magnetic field parallel to the interlayer as a function of two variables, namely temperature and applied voltage. We will start with the standard expression

$$I = e\sum_{kp}[W_{kp}n_k(1-n_p) - W_{pk}n_p(1-n_k)] \tag{36}$$

relating the current in the tunnel junction to the probabilities of transitions W_{kp} and W_{pk} between electron states k and p of the left and right leads of the junction, respectively. The dependence of the current on the voltage applied to the junction is taken into account in (36) in the Fermi filling numbers $n_k = n_F(E_k - eV), n_p = n_F(E_p)$.

The processes of magnetic barrier release occurring due to scattering by phonons are considered in transition probabilities W_{kp}. As in section 3.1, transition amplitude T_{kp} may be calculated in the second order of the perturbation theory in the impurity potential in the leads, but in an arbitrary order with respect to the electron–phonon interaction. Then for the impurity-averaged square of the transition amplitude with absorption of phonons $q_1, \ldots, q_m$ and emission of phonons $q'_1, \ldots, q'_l$ we obtain an expression coinciding with the second term in (32) where the Green function $g(\boldsymbol{r}, \boldsymbol{r}')$ is replaced by the matrix element of the operator Green function:

$$\langle \boldsymbol{r}'; (N_{q_1}-1)\ldots(N_{q_m}-1), (N_{q'_1}+1)\ldots(N_{q'_l}+1)|1/(E_k - \hat{\mathcal{H}}) \times |\boldsymbol{r}; N_{q_1}\ldots N_{q_m}, N_{q'_1}\ldots N_{q'_l}\rangle. \tag{37}$$

Here $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{ph} + \hat{\mathcal{H}}_{e-ph}$ (29), (11), (14). Expanding (37) in a power series of $\hat{\mathcal{H}}_{e-ph}$, it can be verified easily that (37) coincides (within the accuracy of insignificant pre-exponential factors) with the matrix element of the $\hat{T}$-operator (12) for the transition between states localized on impurity centres situated at points $\boldsymbol{r}$ and $\boldsymbol{r}'$. Hence, calculating the current using (36), we will use for W_{kp} the expressions obtained in section 2 for the probability $W_{1\to 2}$ of transition between localized states. In doing so, energies E_1 and E_2 should be substituted by E_k and E_p and d should mean interlayer thickness rather than centre spacing.

3.2.1. Non-linear differential conductance at $T = 0$. Let us calculate the conductance, $G(V)$, of the tunnel junction at zero temperature when, besides elastic tunnelling, only phonon emission processes are possible. Differentiating (36) at $T = 0$ with respect to V, we obtain

$$G(V) = e^2\sum_{kp} W_{kp}(1-n_p)\delta(E_k - eV). \tag{38}$$

As shown in section 2, the hoppiing probability increases exponentially with an increase in the energy difference between the initial and final states due to a greater shift x_0 of the oscillator centre upon phonon emission. Since the maximum value of energy difference $E_k - E_p$ in (38) is eV, the estimation

$$G_{in}(V) \propto W_{kp|E_k-E_p=eV} \propto \sum_{l=1}^{\infty} \alpha^{2l} \exp\left\{-\frac{\lambda^2}{2l}\left(\frac{eV}{\hbar s}\right)^2 - \frac{1}{2\lambda^2}\left(d - \lambda^2\frac{|eV|}{\hbar s}\right)^2\right\} \tag{39}$$

is valid for the inelastic contribution to conductance within an exponential accuracy (20). The total conductance, $G(V)$, is the sum of inelastic and elastic contributions; the latter, as shown in section 3.1, taking the form $G_{el} \propto \exp(-d^2/2\lambda^2)$ and being free of an exponential voltage dependence. At voltages $|eV| > 2(\hbar s/d)\ln(1/\alpha)$ the inelastic contribution exceeds the elastic one and conductance starts to increase exponentially with voltage. As the voltage increases, the processes with emission of 1,2,3,... phonons are sequentially initiated. At voltages greater than $(\hbar s/|e|\lambda)\ln^{3/2}(1/\alpha)$ dependence (39) has an asymptotic form

$$G_{in}(V) \propto \exp\{-2\lambda(|eV|/\hbar s)\ \ln^{1/2}(1/\alpha) - (1/2\lambda^2)(d - \lambda^2|eV|/\hbar s)^2\}. \tag{40}$$

The exponential increase of conductance continues up to voltage $|eV^*| = \hbar sd/\lambda^2$ at which the processes with oscillator centre shift $x_0 = d$ are allowed and estimation (5) is valid for conductance.

3.2.2. Temperature dependence of linear conductance. Let us calculate the temperature dependence of conductance $G(T)$ in the linear regime, $eV \ll T$ [5]. Differentiating current (36) with respect to V in this limit, we find

$$G(T) = \frac{e^2}{T}\sum_{kp} W_{kp} n_k (1 - n_p). \tag{41}$$

Expanding hopping probability W_{kp} in a power series of the electron–phonon interaction constant, linear conductance may be presented as

$$G(T) = G_{el} + G^{(1)}(T) + G^{(2)}(T) + \dots \tag{42}$$

where $G^{(1)}, G^{(2)}, \dots$ are the contributions to conductance caused by the processes involving 1,2,... phonons. At a small electron–phonon interaction constant α (and not too large interlayer thickness d) the temperature dependence of linear conductance is determined by several corrections in (42). Using (16) for the hopping probability with the emission of one phonon and relationship (24) from (41) we obtain

$$G^{(1)}(T) \propto \begin{cases} \alpha^2 \exp(-\frac{d^2}{2\lambda^2}) & L_T > d \\ \alpha^2 \exp(-\frac{d^2}{2\lambda^2} + \frac{(d-L_T)^2}{4\lambda^2}) & L_T < d. \end{cases} \tag{43}$$

From (43) it is seen that at $T < \hbar s/d$ no effects of magnetic barrier release are exhibited. This is due to the fact that the tunnelling processes involving phonons with large momenta are suppressed at low temperatures.

Substituting (17) and (18) into (41), we find the second correction:

$$G^{(2)}(T) \propto \begin{cases} \alpha^4 \exp(-\frac{d^2}{2\lambda^2}) & L_T > 2d \\ \alpha^4 \exp\left(-\frac{d^2}{2\lambda^2} + \frac{(d-L_T/2)^2}{3\lambda^2}\right) & L_T < 2d. \end{cases} \tag{44}$$

Note that the exponential increase of correction $G^{(2)}$ starts at threshold temperature

$$T_2 = \hbar s/2d \tag{45}$$

when correction $G^{(1)}$ is still independent of temperature. Therefore, both corrections (43) and (44) should be considered when investigating the temperature dependence of linear conductance at small α.

If constant α is not too small, inelastic processes involving 3,4,5,... phonons are initiated with increasing temperature. At sufficiently high temperature, when $T - T_2 \gg T_2(\lambda/d)\ \ln^{3/2}(1/\alpha)$, conductance is determined by multiphonon processes and its temperature dependence found by means of (23), (41) takes the form

$$G(T) \propto \exp\left[-(d/\lambda)((L_T/2\lambda) + 2\ln^{1/2}(1/\alpha)) + (L_T^2/8\lambda^2) + (L_T/\lambda)\ \ln^{1/2}(1/\alpha)\right]. \tag{46}$$

At $T \sim (\hbar s/\lambda)\ \ln^{1/2}(1/\alpha)$, the $G(T)$ dependence is saturated at value (5).

3.2.3. Non-linear conductance at finite temperatures. In sections 3.2.1 and 3.2.2 conductance was studied at $T = 0$ and $T \gg |eV|$. It is of interest, however, that at not very high voltages ($V < \hbar s d/|e|\lambda^2$) a pronounced temperature dependence also exists within the temperature range $\hbar s/d < T \ll |eV|$. Using (16) and (36) for a single-phonon contribution to conductance at (T and V different from zero), we find

$$G^{(1)}(T,V) \propto \begin{cases} \alpha^2 \exp\left\{-\frac{\lambda^2(eV/\hbar s)^2}{2} - \frac{\left(d-\lambda^2(|eV|/\hbar s)\right)^2}{2\lambda^2}\right\} & L_T > d - \frac{2\lambda^2|eV|}{\hbar s} \\ \alpha^2 \exp\left\{-\frac{d^2}{2\lambda^2} + \frac{\left(d-L_T\right)^2}{4\lambda^2}\right\} \exp\left(\frac{|eV|}{T}\right) & L_T < d - 2\lambda^2\frac{|eV|}{\hbar s}. \end{cases} \tag{47}$$

It can be readily seen that at $T = 0$ equation (47) coincides with the first term of series (39) and at $V = 0$ it coincides with (43). The exponential temperature dependence within the range $T \ll |eV|$ is conditioned by the fact that at $T > 0$ tunnelling processes involving emission of a phonon with energy exceeding $|eV|$ are allowed; this brings about a shift of the oscillator centre greater than that at $T = 0$.

For multiphonon contributions to conductance, the temperature dependence at $T \ll |eV|$ is largely due to the possibility of increasing the total shift of the oscillator centre at the expense of the combined processes involving both emission and absorption of phonons. For instance, for the contribution of combined double-phonon processes to conductance, from (18) and (36) we obtain

$$G^{(2)}(T,V) \propto \begin{cases} \alpha^4 \exp\left\{-\frac{\lambda^2}{2}\left(\frac{eV}{\hbar s}\right)^2 - \frac{(d-\lambda^2\frac{|eV|}{\hbar s})^2}{2\lambda^2}\right\} & L_T > 2d - 3\lambda^2\frac{|eV|}{\hbar s} \\ \alpha^4 \exp\left\{-\frac{d^2}{2\lambda^2} + \frac{(d-L_T/2)^2}{3\lambda^2} + \frac{|eV|}{2T} - \frac{\lambda^2}{4}\left(\frac{eV}{\hbar s}\right)^2\right\} & L_T < 2d - \frac{3\lambda^2|eV|}{\hbar s} \\ & |eV|T < (\frac{\hbar s}{\lambda})^2. \end{cases} \tag{48}$$

At $V = 0$ equation (48) coincides with (44). At low temperatures, $T < \hbar s/d$, besides (48), account should also be taken of the contribution made by the processes involving the emission of two phonons (described by the second term of series (39)). Note that the temperature-dependent double-phonon contribution (48) starts increasing abruptly at lower temperatures than the single-phonon contribution (47). Hence, generally speaking, both contributions should be taken into account even at small α.

For the case where the electron–phonon interaction constant is not too small, combined processes involving emission and absorption of a large number of phonons become important with increasing temperature. Their contribution to conductance may be found by means of (23) and (36) and has the form

$$G(V,T) \propto \exp\{-(d/\lambda)((L_T/2\lambda) + 2\ \ln^{1/2}(1/\alpha)) + (L_T^2/8\lambda^2) + (L_T/\lambda)\ \ln^{1/2}(1/\alpha)\}\exp(|eV|/2T). \tag{49}$$

Formula (49) is valid at $d - |eV|\lambda^2/\hbar s - L_T/2 \gg \lambda\ \ln^{3/2}(1/\alpha), T \ll (\hbar s/\lambda)\ln^{1/2}(1/\alpha)$. At $T \sim (\hbar s/\lambda)\ln^{1/2}(1/\alpha)$ conductance is saturated at value (5) irrespective of applied voltage. Combined processes are insignificant in the region of the values of V and T such that $d - |eV|\lambda^2/\hbar s - L_T/2 < 0$. In this case conductance is determined by the tunnelling processes involving emission of phonons and is described by (41).

4. Phonon-assisted transitions between the edge states in 2D ballistic structure

Buttiker's approach [7], based on the consideration of edge states, has been widely used to describe the experiments on the integer quantum Hall effect. In [8, 9] it has been shown that electron transitions between different edge states may lead to deviation of the Hall resistance from the quantized value. Two situations are possible: (i) transitions occur between two different edge states located near to one sample boundary and (ii) transitions between the edge states located near opposite sample boundaries. Numerous experiments on measuring the length of time to establish equilibrium between initially non-equilibrium populated edge states correspond to the first case [10, 11]. If the confining potential is fairly smooth, then the distance d between two neighbouring edge states may appear to be much larger than the magnetic length (for instance, estimation $d/\lambda \simeq 5.2$ was obtained in [10]). In the second case, when transitions between states located at different edges of the sample are considered, d is determined by the sample width and, hence, it is also large compared to λ. It would be expected that for $d \gg \lambda$ in a sufficiently pure sample transitions between edge states are accomplished mainly due to the processes involving a single or several phonons.

In this section we calculate to an exponential accuracy the current passing between two edge states as a function of temperature T and the difference of their electrochemical potentials $\Delta\mu$ (the current being due to the scattering processes involving one or two phonons). It is assumed that impurities are absent so that the wavevector along the boundary is conserved.†

The contribution to current caused by single-phonon processes is determined by the expression

$$I_1 = J_1 \exp(-d^2/2\lambda^2)\exp\left\{-(\frac{\hbar s d}{\lambda^2} - |\Delta\mu|)/T\right\}. \tag{50}$$

† In the case of rough boundaries, the problem on scattering between edge states located near opposite sample boundaries is reduced to the problem considered in section 3.

At $\Delta\mu \to 0$ this expression was obtained in [12, 13], where pre-exponential factor J_1 (proportional to the square of the electron–phonon interaction constant) was also calculated. In (50) it is assumed that $d \gg \lambda, |\Delta\mu| < \hbar sd/\lambda^2$ and $T \ll \hbar sd/\lambda^2$. The occurrence of the exponential temperature dependence in (50) is related to the fact that owing to the conservation of the wavevector along the boundary upon scattering, the main contribution to current is made by the processes involving emission of phonons with energy $\hbar sd/\lambda^2$. The last multiplier in (50) describes the small probability of finding the unoccupied final state whose energy is below the corresponding electrochemical potential by $(\hbar sd/\lambda^2 - |\Delta\mu|)$. At $|\Delta\mu| > \hbar sd/\lambda^2$ equation (50) is free of the last multiplier and spontaneous phonon emission is possible at any temperature [10].

Two contributions to current related to double-phonon processes may be discerned. The first is due to scattering between edge states involving emission of two phonons and takes the form

$$I_2 = J_2 \exp(-d^2/4\lambda^2) \exp\left\{-\left(\frac{\hbar sd}{\lambda^2} - |\Delta\mu|\right)/T\right\}. \tag{51}$$

This expression is quite analogous to (50) but distinguished from it by pre-exponential factor J_2, which is proportional to the fourth power of the electron–phonon interaction constant, and a less pronounced dependence on d associated with a more effective suppression of the magnetic barrier. The second contribution is related to the combined processes when one phonon is absorbed and the other is emitted, and takes the form

$$I_2' = J_2' \exp(-d^2/4\lambda^2) \exp(-\hbar sd/2\lambda^2 T) F(\Delta\mu, T) \tag{52}$$

$$F(\Delta\mu, T) = \begin{cases} \exp\left\{\frac{|\Delta\mu|}{2T} - \frac{\lambda^2(\Delta\mu)^2}{4(\hbar s)^2}\right\} & |\Delta\mu|T < (\hbar s/\lambda)^2 \\ \exp\left\{\frac{(\hbar s)^2}{4\lambda^2 T^2}\right\} & |\Delta\mu|T > (\hbar s/\lambda)^2 \end{cases}$$

Comparison of (50) and (52) in the simplest case ($|\Delta\mu| \to 0$) shows that the corresponding exponential temperature dependences are significantly different: the activation energy corresponding to the double-phonon processes is half as large. This is due to the fact that the main contribution to current is made by the combined processes in which the wavevectors of both the emitted and the absorbed phonons are equal in magnitude to $d/2\lambda^2$ so that the temperature dependence of the contribution to the current (52) at $|\Delta\mu| \to 0$ is governed only by the small probability of absorption of a phonon with energy $\hbar sd/2\lambda^2$. Therefore, measurement of the activation energy in the temperature dependence of the inverse length of edge state population relaxation at $|\Delta\mu| \ll T$ may be recommended for experimental observation of double-phonon processes. The estimations show, in the case of GaAs (and also when considering pre-exponent factors for the deformation potential interaction), that one- and two-phonon contributions to the scattering rate between edge states related to the zeroth and first Landau levels for $\mathcal{H} = 3.7$ T become equal at $T \approx 0.8$–1.6 K for $d/\lambda = 4$–5.

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